Study of the Electron Beam Transported in Sectional Drift Tubes[1]

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Abstract – Theoretical and experimental investigations of an annular magnetized electron beam transported in two-section drift tubes have been conducted. It has been shown that when the lengths of the tube sections are far beyond their radii the current and the state of the beam in the drift channel correspond to the current and state of the beam in infinite homogeneous drift tubes. The values of these currents for the stationary states have been determined based on laws of conservation.

It has been demonstrated for the first time that when the injected beam current I_{inj} exceeds the limiting transport current I_{lim2} for the wider drift section the transmitted beam current decreases abruptly due to the formation of a virtual cathode at the tube junction and to accumulation of the space charge in this region.

1. Introduction

In most of relativistic high power microwaves devices, high-current electron beams are transported in vacuum waveguides composed of sections of different radii. In this connection, it is essential to determine the values of the limiting transport currents and the states of the beam in such systems upon the formation of a virtual cathode (VC) [1].

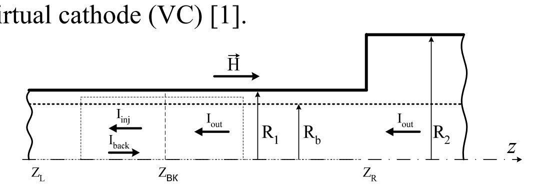

Fig. 1. Schematic of the two-section drift tube bounded by foils. R_1, R_2, L_1, L_2– radii and lengths of the section, R_b – beam radius; I_{out} I_{inj}, I_{back} – output, injection and reflected from VC currents recpectively; the dashed line stands for the region in the narrower section where the field distribution is independent of the z coordinate

At present the problems of beam transportation is mainly solved by numerical simulation with the use of universal PIC-based electrodynamic codes, such as LSP and KARAT [2, 3]. The behavior of annular electron beams transported vie sectional drift tubes in a strong magnetic field has been much addressed in the literature [4-6]. However, many fundamental theoretical problems of transportation of such beams, even in the simplest drift systems, are as yet unsolved [7-9].

This paper reports on theoretical and experimental studies of the magnetized electron beam transported in a multisectional drift tube in the steady states.

2. Theory

A model problem of the beam transport theory is determination of the electron beam currents in a homogeneous drift tube. For stationary states of the electron beam, this problem can be solved based on the laws of conservation of energy and z-component of the field and particle momentum [10]. Let a thin annular electron beam be transported in a homogeneous drift tube. The electron beam is injected at the left. The tube has the electrostatic potential of an anode and is placed in a strong longitudinal guiding magnetic field of strength $\vec{H}$ parallel to the tube axis (Fig.1). We determine the stationary states of the beam in an infinite homogeneous drift tube (*L*>>*R*) without studying their stability and ways of realization and demonstrate that the beam in the tube may occur in the state with VC other than passing homogenously through the tube.

Assume that in the plane z = z_{VC} an azimuthally symmetric VC is formed. We elucidate what current passes behind the VC when the beam is stationary. Select a volume (with the boundary shown dashed in Fig. 1) such that the field and beam parameters at its left and right boundaries can be considered as independent of the coordinate z. From the law of conservation of energy we have:

$$I_{out} = I_0 \left(\Gamma - \gamma_R\right)\left(\gamma_R^2 - 1\right)^{1/2} \times \left[2\gamma_R \ln(R_a / R_b)\right]^{-1}, \qquad (1)$$

$$I_{inj} + I_{back} = I_0 \left(\Gamma - \gamma_L\right)\left(\gamma_L^2 - 1\right)^{1/2} \times \left[2\gamma_L \ln(R_a / R_b)\right]^{-1}, \qquad (2)$$

where I_0 = mc^3/e, I_{inj}, I_{out}, I_{back} are, respectively, the injected current, the transmitted current and the current of electrons reflected form the VC, $\Gamma = 1 + eU/mc^2$, $\gamma_{R,L} = e\varphi_{R,L} / mc^2$, U, $\varphi_{R,L}$ is the electrostatic potentials of the tube (anode) and beam at the right (R) and left (L) boundaries, *m* is the mass of an elec-

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tron, e is the electric charge of an electron, and *c* is the velocity of light in vacuum.

The factor of beam current reflection from the VC is expressed as $\alpha = I_{back} / I_{inj}$. Since $I_{out} = I_{inj} - I_{back}$, we have $0 \le \alpha \le 1$ and $I_{inj} + I_{back} = I_{out}(1+\alpha)(1-\alpha)^{-1}$ and, taking into account (1) and (2), we obtain:

$$(1-\alpha)(\Gamma-\gamma_L)\left(\gamma_L^2-1\right)^{1/2}\left[\gamma_L(1+\alpha)\right]^{-1} = \tag{3}$$
$$= (\Gamma-\gamma_R)\left(\gamma_R^2-1\right)^{1/2}\left[\gamma_R\right]^{-1}$$

The additional relation between γ_L and γ_R is derived from the law of conservation of the z-component of the field and particle momentum in the marked volume.

$$\left(\Gamma-\gamma_R\right)^2 + 2\left(\Gamma-\gamma_R\right)\left(\gamma_R^2-1\right)\left[\gamma_R\right]^{-1} = \tag{4}$$
$$= \left(\Gamma-\gamma_L\right)^2 + 2\left(\Gamma-\gamma_L\right)\left(\gamma_L^2-1\right)\left[\gamma_L\right]^{-1}$$

With the expression $f(\gamma) = (\Gamma-\gamma)^2 + 2(\Gamma-\gamma)(\gamma^2-1)\gamma^{-1}$, relation (4) takes the form $f(\gamma_R) = f(\gamma_L)$. This equation, exclusive of the trivial solution $\gamma_R = \gamma_L$, has solutions at $\gamma_R \ne \gamma_L$, also. Actually, the function f has a maximum at $\gamma = \Gamma^{1/3}$ (Fig.2) and $f_{\max} = (\Gamma^{2/3}-1)^2(\Gamma^{2/3}+2)$, and for $(\Gamma-1)^2 \le f \le f_{\max}$ the relativistic factor γ has two values. At $f = (\Gamma-1)^2$, $\gamma = 1$ and $\gamma = \gamma_F = -0.5 + \sqrt{2\Gamma + 0.25}$, where γ_F is the relativistic factor of the beam with the so-called Fedosov current I_F obtained for a thin electron beam in a coaxial diode with magnetic insulation (CDMI) [11]. In so doing, the electron flux was naturally taken to follow one direction, from the cathode, and $I = I_F$. In the general case, for each specific γ the current in the system can assume any value in the range $-I_F \le I \le +I_F$, since the flux of the z-component of the field and particle momentum does not depend on the sign of electron velocity. In the event that all electrons are reflected from the VC ($\alpha = 1$), the transmitted current in the tube is equal to zero ($I_{out} = 0$). This means that the beam charge density at the right equals the charge density at the cathode in CDMI and at the left the Fedosov current is divided in two, i.e., a beam with a current $I_F / 2$ streams rightward and an identical beam streams leftward.

Current (1) has a maximum (the limiting current) at $\gamma_1 = \Gamma^{1/3}$. A plot of the current is shown in Fig.2. It turns out that to satisfy equation (3), γ_R must lie on the left branch of the curve of the current flow path in the tube for a thin annular beam (Fig.1). If so, $0 \le \alpha \le 1$ which is consistent with the physical meaning of the reflection factor. This state corresponds to the "squeezed" state of the beam considered in [4, 6, 7, 10].

Figure 3 shows the α dependence of $I_{out}\ln(R_a/R_b)$ at $\Gamma = 2$. It is seen from this figure that the current behind the VC can take values from zero to I_{lim} for a homogeneous drift tube. Since the current I_{out} is unambiguously related to the reflection factor α, the direct (injected) current I_{inj} and the current reflected from the VC I_{back} for each I_{out} are also uniquely determined, i.e., the current behind the VC uniquely determines the state of the system in the steady case.

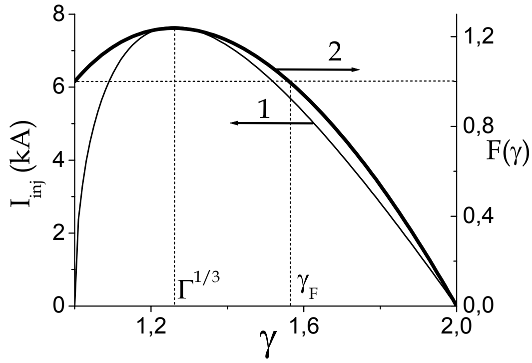


Fig. 2. Dependence of beam injection current (1) and F(γ) (2) on γ relativistic factor of electron. $2\ln(R_a/R_b) = 1$. $\Gamma = 2$

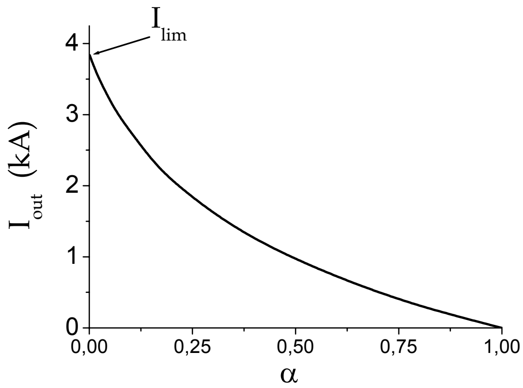


Fig. 3. Transmitted beam current I_{out} vs the reflection coefficient α. $2\ln(R_a/R_b) = 1$. $\Gamma = 2$

The specific value of I_{out} is given by the boundary conditions of transportation. As shown in [10], in an infinite homogeneous tube when $\alpha = 0$, all electrons passes through the VC without reflection and the beam current is equal to the limiting current. In the case where a long homogeneous tube for which $L_1 \gg R_1$ is joined with a tube of larger radius ($L_2 \gg R_2$, $R_1 < R_2$), the currents in the system are determined by the limiting current in the tube of larger radius $I_{out} \approx I_{lim2}$.

Consider a sectional tube with two sufficiently long sections ($L_1 \gg R_1$, $L_2 \gg R_2$) such that they can be taken infinite and we can apply to them the results reported above. In solving the problems on the electron beam current in a two-section drift tube with VC, it is essential to determine the injected current I_{inj} at which the VC formed at the tube junction starts moving to the injection region at the beginning of the first tube section (Fig.1). In [7] this current is called the transition current I_{Tr}. The value of I_{Tr} can be found within the framework of two models.

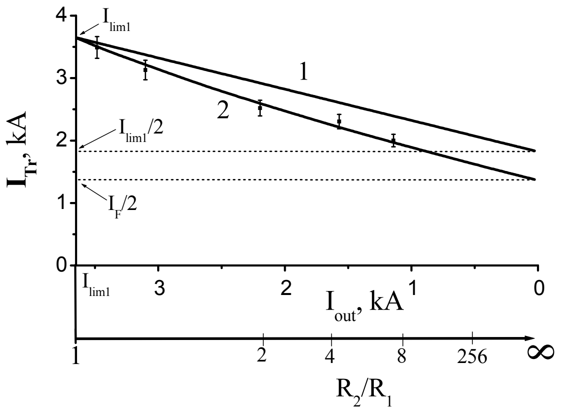


Fig. 4. Plot of the VC transition current I_{Tr} versus output current for R_b=3.5mm, R_1=10mm. Γ = 2. Curve $(I_{lim1}+I_{lim2})/2$ (1) and the results of theoretical calculations (2). Black points showed the result of computer simulation using PIC code KARAT

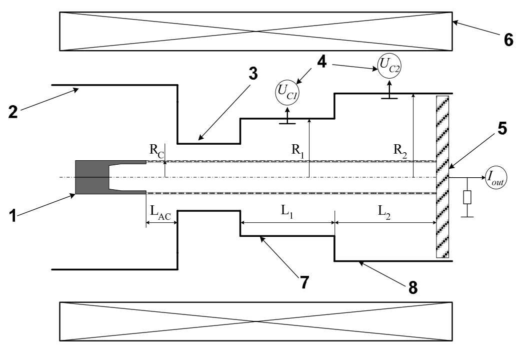


Fig. 5. Schematic diagram of the experimental arrangement: (*1*) cathode; (*2*) anode tube; (*3*) anode insert; (*4*) capacitive voltage dividers; (*5*) collector; (*6*) solenoid; (*7*, *8*) drift channel sections of small and large radius, respectively

In the simplest model of the limiting charge, it is assumed that the VC starts to shift when the charge density in both sections reaches the limiting value, which is equal to the density corresponding to the limiting currents [4]. In this case, $I_{lim1} = I_{Tr} + I_{back}$, $I_{lim2} = I_{Tr} - I_{back}$ and

$$I_{Tr} = (I_{\text{lim}1} + I_{\text{lim}2})/2 \qquad (5)$$

In [7] the values of I_{Tr} were found by analyzing the flux of the z-component of the field momentum through the face surface of a two-section drift tube against the injected current. In so doing, it was assumed that the maximum value of the flux corresponds to the passage of the VC into the injection region and to the “squeezed” state of the beam and the flux remains constant with increasing injection current. In effect, this assumption implies that the transition current corresponds to the steady-state value of the injected current for a infinite homogeneous tube of radius R_1 at $I_{out} = I_{lim2}$. This model stems from the fact that if the condition ($L_1 >> R_1$, $L_2 >> R_2$) is fulfilled the sectional tube can be considered as a sequence of infinite homogeneous tubes.

The model of superposition of infinite homogeneous tubes has been tested by simulating the transportation of a thin annular electron beam in a two-section drift tube with the use of the PIC code КАРАТ [3]. In all computations, the radii of the narrow tube section and of the beam remained constant (R_1 = 1 cm, R_b = 0.35 cm) and the radius of the wider tube section was varied in the range from 2 cm to 10 cm. The calculations support that as the injected current reaches the limiting current for the second tube section $I_{inj} = I_{lim2}$, a VC occurs in the plane z_{12} [4,6]. The values of the transition current I_{Tr} obtained in the simulation (points in Fig. 4) are invariably lower than $I_{inj}^{\max} < (I_{\text{lim}1} + I_{\text{lim}2})/2$ (curve 1 in Fig. 4) and agree well with the analytical calculations based on the model of stationary states (Curve 2 in Fig. 4).

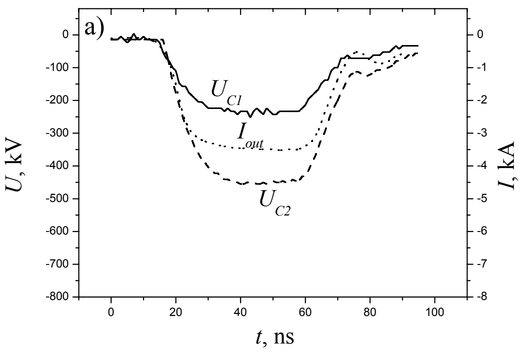
The model assumes that the lengths of the tube sections are much larger than their radii. In this event, the geometry of the system to the left of the VC approximates that of the infinite smooth tube considered above, and the tube section with R_2 on the right does no more than specifies the value of the transmitted current. Note that this model ignores the change in the flux of the z-component of the field and particle momentum through the face surface at the tube junction when the VC occurs and changes its position in response to an increase in injected current. This may cause a change in the values of the currents in this system.

3. Experiments and numerical simulation

The theoretical results and the model notions have been tested in experimental studies and numerical simulation of the electron beam transported in sectional tubes.

The experimental investigation were conducted on the high-current electron accelerator SINUS-7 [12] operating at a diode voltage of up to 2 MV, a diode current of up to 20 kA, and a current pulse duration of 50 ns. A schematic of the experimental arrangement is shown in Fig. 5. The electron beam was generated in a vacuum diode with magnetic insulation and was injected into a two-section drift tube vie an anode insert. Electrons were emitted from a cylindrical graphite explosive-emission cathode of radius $R_C \approx 9.5$ mm and ridge width 0.5 mm. An annular electron beam was thus formed in a homogeneous longitudinal magnetic field of strength ~ 15 kOe. The radius and the length of the anode insert were 20 mm and 130 mm, respectively.

The lengths of the drift tube sections ($L_{dr1} \approx 150$ mm, $L_{dr2} \approx 400$ mm) were far beyond their radii ($R_{A1} \approx$ 24 mm, $R_{A2} \approx 41$ mm). The current injected into the drift tube was varied by varying the distance between the cathode and the anode insert (L_{AC}), with the diode voltage kept constant $U \approx -(800 \pm 30)$ kV. The potential difference between the electron beam and the drift tube was measured with capacitive dividers *4* arranged in the central part of each tube sections. The beam current was measured with a low-inductance ohmic shunt brought into collector circuit *5*.



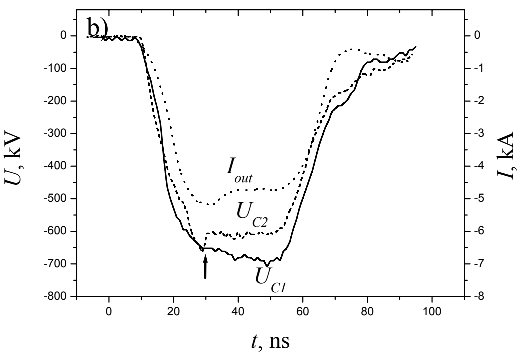


Fig. 6. The typical oscillograms of the beam potential and current in the two-section drift channel (a) without and (b) with VC (arrow indicates the moment of VC formation)

The vacuum diode current I_D was measured in a series of experiments in which the collector was arranged at the end of the anode insert. In this geometry, varying the distance L_{AC} from 70 mm (the cathode occurs in the anode tube, Fig.5) to -15 mm (the front edge of the cathode protrudes 15 mm into the anode insert) causes the diode current to vary within $I_{FD} < I_D < I_{FA}$, where $I_{FD} \approx 3.0$ *kA* and $I_{FA} \approx 6.8$ *kA* and are the Fedosov currents of coaxial diodes with magnetic insulation which are determined by the radii of the anode tube and anode insert, respectively.

In experiments with the electron beam transported in the two-section drift tube when the collector was arranged at the end of the system (Fig.5), the injected current upon the VC formation was determined with account of the reflected current I_{back}: $I_{inj} = (I_D + I_{out})/2$, where $I_D = (I_{inj} + |I_{back}|)$ and is the experimentally measured value of the diode current without VC for the given position of the cathode L_{AC}.

Oscillograms

Figure 6 shows typical waveforms of the signals measured with the capacitive dividers and shunt for the case of the beam transportation with and without VC. With large gap widths L_{AC}, when $I_D < I_{lim2}$ and the VC is absent, the oscillograms of the signals have a near-trapezoidal shape (Fig.6a). The transmitted current I_{out}, in this case, coincides with the injected current $I_{out} = I_{inj} = I_D$ (straight segment in Fig.6a). With injected currents higher than the limiting current for the second tube section $I_{inj} > I_{lim2}$, one can distinguish the moment on the U_{C2} oscillograms after which U_{C2} shows an abrupt decrease (marked by the arrow in Fig.6b). The results of CARAT simulation of the beam transportation in the experimental conditions have shown that this moment corresponds to the VC formation. Therefore, the experimental data were processed on the assumption that the value of I_{out} after this moment and after the decrease in this current corresponds to the current behind the VC.

Transport current

The results of experiments and numerical simulation (Fig.7a) have shown that the VC is formed at a distance of order (R_2 - R_b) from the tube junction (point *B* in Fig.7) when the injected current slightly exceeds I_{lim2}. As this takes place, the transmitted current decreases abruptly, $I_{out} \approx I_{lim2} - \Delta I$. This decrease in transmitted current is due to the fact that at the instant the VC is formed the flux of the z-component of the generalized momentum changes. The change of the flux for the case under consideration is estimated to correspond to $\Delta I_{max} \approx 0.1 I_{lim2}$ that agrees with the experimentally measured value.

As shown by the results of simulation, the value of ΔI after the VC formation is determined by the VC position relative to the plane of the tube junction. An increase in injection current causes the VC to change its position and to get closer to this plane. At $I_{inj} \sim 1{,}2\ I_{lim2} \approx I_{Tr}$, when the injected current reaches the transition current (point C in Fig. 7), the VC shifts toward the injection plane. The change in the generalized momentum at the expense of the space charge of electrons in the VC becomes insignificant, $\Delta I \rightarrow 0$, and the transport current tends to the limiting value for the section of larger diameter $I_{out} \rightarrow I_{lim2}$.

Potentials and states of the beam

The state of the beam in the drift channel can be judged from measurements of the beam potential. As can be seen from Fig.7a, the beam potential in the drift section of larger radius, U_{C2}/U, and its dependence on the injected current correspond to a beam with the limiting current, that is , to a beam in the "fast" state.

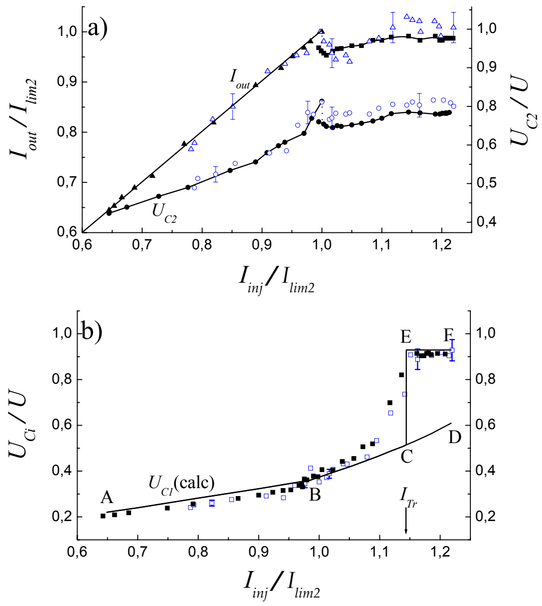


Fig. 7. Plots of the relative beam potential and current versus relative injected current for a two section drift channel. Solid curves in (a) and filled symbols present the results of numerical simulations performed using the PIC code KARAT; open symbols present the experimental data obtained on a SINUS-7 accelerator; curves *ABCD* and *ABCEF* in (b) show the results of approximate calculations using two models (see the text)

The experiments and the simulation have disclosed that as the injected current is increased, the beam potential in the tube section of smaller radius, U_{C1}/U, shows a monotonic increase and once the injected current reaches a certain value, it ceases to grow and remains constant (Fig.7b). To interpret the experimental data, we have estimated the U_{C1}/U ratio from the value of the beam charge in the tube section of smaller radius for two cases (Fig.7b). In the first case, it was assumed that when the injected current exceeds the limiting current I_{lim2}, a VC is formed at the tube junction and a reflected current arises, but the beam is still in the "fast" state (curve ABCD in Fig.7b). In the second case, it was assumed that when the injected current reaches the transition current $I_{inj} = I_{Tr}$ [7], the VC shifts from the region of the tube junction to the beginning of the first tube section, leaving the electron beam in the "squeezed" state behind (curve ABCEF in Fig.7b). It can be seen from this figure that the results of calculations for the second case show a rather good agreement with the experimental data. Hence, in all experiments in which the injected current reached or exceeded the transition current $I_{inj} \geq I_{Tr}$, the electron beam in the tube section of smaller radius occurred in a single-flux "squeezed" state and its potential, as revealed by measurements (Fig.7b), remained constant and invariant with the injected current. The kinetic energy of electrons in the squeezed beam $mc^2(\gamma_b-1) \approx$ (70-80) *keV* was about an order of magnitude lower than their energy on the collector, and the relativistic factor ($\gamma_b \approx 1.15$) was smaller than that of the beam with the limiting current ($\Gamma^{1/3} \approx 1.36$).

Note that the transition current calculated based on the law of conservation of the z-component of the field and particle momentum, I_{Tr} = 5.4 kA, and equal to the current I_{inj} for the stationary state of the beam in an infinite homogeneous tube of radius R_1 at $I_{out} = I_{lim2}$ agrees closely with the minimum experimental injected current at which the beam potential becomes equal to that for a beam in the "squeezed" state. This agreement confirms the applicability of the model of superposition to analyzing the electron beams transported in sectional drift tubes.

The results of simulation have shown that in the range of injected currents $I_{lim2} < I_{inj} < I_{Tr}$ (whereby the VC resides at the tube junction), the beam in the first tube section occurs in a two-flux state and the VC is formed within the diode voltage pulse risetime. Therefore, the low-energy electrons reflected from the VC are accumulated in the drift channel, rather than driven back to the cathode, which leads to an increase in electron density and to a decrease in beam potential. This presumably accounts for the discrepancy between the results of measurements and calculations for the considered range of I_{inj} (Fig.3b, curve BCE), in which this effect was not taken into account.

4. Conclusion

Thus, the results of experimental studies and numerical simulation of electron beams transported in sectional drift tubes have demonstrated that, when the lengths of the drift sections are far beyond their radii, the currents in the drift system correspond to the currents in an infinite homogeneous drift tube of smaller radius in the stationary states. The values of these currents have been determined based on the laws of conservation: the output waveguide prescribes the value of the transmitted current $I_{out} \approx I_{lim2}$, and the injected and reflected currents lie in the range $I_F/2 \leq I_{inj} \leq I_{\text{lim}1}$, $0 \leq I_{back} \leq I_F/2$. The VC resides in the tube section of smaller radius near the beam injection region and the beam behind the VC occurs in the single-flux squeezed state, in which the relativistic factor of the beam is smaller than that for a beam with the limiting current $\gamma_b < \Gamma^{1/3}$, which is supported by the experimental measurements.

It has been shown for the first time that when the injected current I_{inj} exceeds the limiting transport current I_{lim2} for the tube section of larger radius the transmitted current decreases abruptly, which is due to the formation of a virtual cathode at the tube junction and to accumulation of the space charge in this region.

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