Propagation of Apertured Beam of Charged Particles

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Abstract – The problem of calculation of charged-particle beam parameters after transit through an orifice, intercepting part of the beam is considered. A calculation method using an application of the phase-density formalism and relating characteristics of the having passed beam to its structure at the starting section is proposed. Important beam parameters such as radial distribution of density and shape of the envelope are calculated. The cases of beam transportation in free space (an analog of propagation of neutral particles beam) and in uniform magnetic field have been demonstrated by examples of model phase densities.

1. Introduction

Transportation of charged-particles beams in technological facilities is, as a rule, described by their envelopes. “Emittance diagrams” that are projections of the beam phase density in the phase plane, have been of widespread occurrence in the envelope method [1-4]. According to the Liouvill theorem, the area of the diagram (or emittance) is an invariant of ensemble of particles in their motion in medium without collisions. The most simplicity and clearness are inherent in models describing the beam transportation in the paraxial approximation, if the emittance diagram has the form of centrally symmetric ellipse in phase coordinates r,r'. The beam propagation is attended with deformation of the emittance diagram without variation in its ellipticity and area.

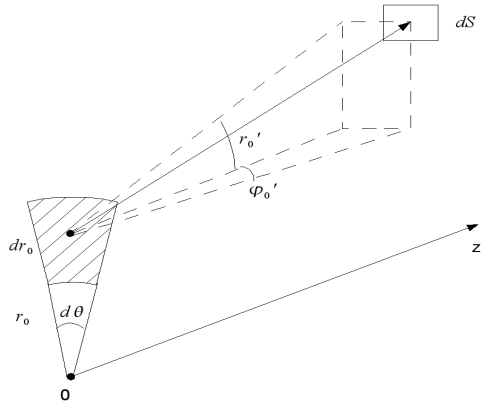
The beam emittance remains invariable with coordinated transportation through a system of channels.

The pattern of transportation becomes more complicated if a partial interception of the beam with channels walls takes place. An essential beam orificing occurs e.g. in devices of concentrated-electron-beam extraction to a dense gaseous medium [5,6]. Minimization of gas inflow thereto from the environment is realized by direct beam burning-through of a number of membranes. Naturally such a method for transportation is accompanied with losses of peripheral particles of the beam, the emittance stops being an invariant. Each membrane burnt-through changes the beam phase diagram and because of this the next membrane is burnt through by the beam with a changed distribution of power density.

This work has to do with the method proposed for description of propagation of the axially symmetric beam of charged particles, that allows the effect of beam orificing to be taken into account. The method is based on using the three-dimensional phase density $f(r,r',\varphi)$ being, in point of fact, an analog of luminance [7]. Here, r – radial coordinate, r' and φ – tangents of slopes of particle trajectories in radial and azimuth directions, respectively. It should be noted the consideration of the dispersion of the azimuth slope removes peculiarities on the axis of the axially symmetric beam, favours adequate description, in particular, gives an insight into a diffusion of orificed beam limits. The description is made in cylindric coordinates.

2. Model of propagation of the orificed beam

Let the phase density of charged particles be equal $f(r_0,r_0',\varphi_0)$ in the orificing plane $(z=0)$. Here index “0” refers to phase coordinates in the reference plane. The number of particles escaping from an elementary area per unit of time, $r_0 dr_0 d\theta_0$ (s. the figure) into the range from r_0' to $r_0'+dr_0'$ and from φ_0 to $\varphi_0+d\varphi_0$, is equal to $f(r_0,r_0',\varphi_0)dr_0'd\varphi_0 r_0 dr_0 d\theta$.



Generally, trajectories of particles, have the form:

$$\begin{aligned} r &= r(r_0,r_0',\varphi_0,z) \\ \theta &= \theta(r_0,r_0',\varphi_0,z), \end{aligned} \qquad (1)$$

where θ – the azimuth coordinate of particle.

At the section z the particles mentioned fall onto the elementary area dS defined by the Jacobian of conversion (1) $dS = Jdr_0'd\varphi_0$, where

$$J = r\left|\frac{\partial\theta}{\partial r_0'}\frac{\partial r}{\partial\varphi_0} - \frac{\partial\theta}{\partial\varphi_0}\frac{\partial r}{\partial r_0'}\right|. \qquad (2)$$

If one succeeds to express r_0', φ_0 has functions of r, r_0, θ, from Eq. (1), the second derivative of the current density at the section z equals:

$$d^2 j = \frac{f(r_0, r, \theta, z) r_0 dr_0 d\theta}{r} \times \left| \frac{\partial \theta}{\partial r_0'} \frac{\partial r}{\partial \varphi_0} - \frac{\partial \theta}{\partial \varphi_0} \frac{\partial r}{\partial r_0'} \right|^{-1}. \quad (3)$$

This relationship establishes a correlation between the phase density of the flow in the reference plane and beam parameters at the arbitrary section. Integrating over the variables r_0 and θ at the surface S_0, located in the plane $z = 0$ and transmitting electrons, we obtain the current density in the plane z:

$$j(r, z) = \int_{S_0} d^2 j,$$

where in the general case S_0 may have any form (sector, segment, concentric slots etc.).

Further, the research is restricted to solid axially symmetric beams, therefore

$$j(r, z) = \int_0^{2\pi} \int_0^b d^2 j, \quad (4)$$

where b – the radius of the orifice aperture.

The proposed calculation algorithm allows determining the distribution of the current density at the section z by beam parameters in the referent plane at known trajectories of electrons. The beam envelope $R(z)$ can be also found from the condition:

$$2\pi \int_0^{R(z)} j(r, z) r dr = 2\pi\kappa \int_0^{\infty} j(r, z) r dr = \kappa I, \quad (5)$$

signifying that within a volume restricted to the surface of revolution $R(z)$, the particle beam is constant and equals to κI, where I – the total current, $\kappa \le 1$.

Obviously, this will be a family of envelops differing in the magnitude κ. It should be noted an envelope is a descriptive characteristic of the beam, however, for calculation of membranes burning through, the knowledge of the flow power density determined by the current density (4) is more important.

Let's consider the method by examples of beams with model phase densities, propagating in free space and uniform magnetic field, where the trajectory equations have the simplest form. The statement of the example pursuens the twofold goal – on the one hand, to demonstrate potentialities of the method, on the other – to present examples for testing of mathematical programs, describing the transportation in more complex cases.

3. The beam in free space

A trajectory of particle in free space in a system of cylindric coordinates is described by a set of equations

$$\begin{cases} r = \sqrt{(r_0 + z r_0')^2 + z^2 \varphi_0^2} \\ \theta = arctg\left[\varphi_0 z / (r_0 + z r_0')\right] \end{cases} \quad (6)$$

with the Jacobian

$$J = z^2. \quad (7)$$

Here the reference of the azimuth coordinate θ is made relative to the position $\theta = 0$ at $z = 0$. Finding from Eq. (6)

$$r_0' = (r\cos\theta - r_0)/z, \quad \varphi_0 = r\sin\theta / z, \quad (8)$$

and substituting them into Eq.(3), one can obtain in terms of (7):

$$d^2 j = z^{-2} \cdot f\left\{ r_0, \frac{(r\cos\theta - r_0)}{z}, \frac{r\sin\theta}{z} \right\} r_0 dr_0 d\theta. \quad (9)$$

Further, with the aid of Eq. (4) and (5), the distribution of the current density and the beam envelope are calculated.

Let's consider the model phase density

$$f(r_0, r_0', \varphi_0) = \begin{cases} \left(1 + \frac{r_0^2}{a^2} + \frac{r_0'^2}{\sigma^2} + \frac{\varphi_0^2}{\sigma^2}\right)^{-2} & at\ r \le b, \\ 0 & at\ r > b, \end{cases} \quad (10)$$

where b – the radius of the orifice aperture, σ and a – angular and spatial dispersions of beam, respectively.

Then Eq. (3) and (4) take the form:

$$d^2 j = \frac{r_0 dr_0 d\theta}{z^2} \times \left[1 + \frac{r^2}{z^2\sigma^2} + \left(\frac{1}{a^2} + \frac{1}{z^2\sigma^2}\right) r_0^2 - \frac{2 r r_0}{z^2\sigma^2}\cos\theta\right]^{-2}, \quad (11)$$

$$j = \frac{\pi a^2}{2z^2} \cdot \frac{1}{1 + a^2/\sigma^2 z^2 + r^2/\sigma^2 z^2} \times \left[\frac{b^2/a^2 + b^2/\sigma^2 z^2 - r^2/\sigma^2 z^2 - 1}{A} + 1\right], \quad (12)$$

where

$$A = \sqrt{\left(\frac{b^2}{a^2} + \frac{b^2}{\sigma^2 z^2} + 1\right)^2 + \frac{2}{\sigma^2 z^2}\left(\frac{b^2}{a^2} - \frac{b^2}{\sigma^2 z^2} + 1\right) r^2 + \frac{r^2}{\sigma^4 z^4}}.$$

The total current $I = \pi^2 a^2 \sigma^2 \ln\left(1 + b^2/a^2\right)$, the beam envelope

$$R(z) = \sqrt{a^2\left(\eta^{\kappa} - 1\right) + \frac{\eta - \eta^{1-\kappa}}{\eta^{1-\kappa} - 1}\sigma^2 z^2}, \quad (13)$$

where $\eta = 1 + b^2/a^2$.

Specifically, with $a \to \infty$ (which signifies that at the starting section the current density is uniform on a radius) Eq. (12) and (13) are simplified:

$$j=\frac{\pi\sigma^2}{2}\left\{\frac{\left(b^2-r^2\right)/\sigma^2z^2-1}{\sqrt{\left[\frac{\left(b^2+r^2\right)}{\sigma^2z^2}+1\right]^2-\frac{4b^2r^2}{\sigma^4z^4}}}+1\right\}, \quad (14)$$

$$R(z)=\sqrt{\kappa b^2+\frac{\kappa}{1-\kappa}\sigma^2z^2}. \quad (15)$$

The expressions (12) and (14) account for a jump-like variation in the current density at the edge of the orifice aperture at $z=0$. However, already for the small z the jump-like character disappears, and j differs from zero with as one likes large r. The beam has no sharp boundaries even near the orifice. The fact that the envelopes are a family of hyperbolas for any orifice draws our attention. This is a feature of the distribution (10). For other phase densities (e.g. appropriate for tubular beams) the envelopes have a more complicated form.

4. The beam in the uniform magnetic field

The radial and azimuth coordinates in the uniform magnetic field follow the law:

$$\begin{cases} r=\sqrt{r_0^2+\left(r_0'^2+\varphi_0^2+2r_0\varphi_0K\right)\frac{\sin^2Kz}{K^2}-r_0r_0'\frac{\sin 2Kz}{K}} \\ \theta=arctg\frac{\varphi_0\sin Kz\cos Kz+r_0'\sin^2Kz}{Kr_0+\varphi_0\sin^2Kz-r_0'\sin Kz\cos Kz}, \end{cases} \quad (16)$$

where $K=\frac{qB}{2mV_{z0}}$, q, m – the charge and mass of particle, respectively, B – the magnetic induction, V_{z0} – the longitudinal component of the particle velocity, taken as being constant.

The Jacobian of the conversion (16) equals

$$J=\frac{\sin^2Kz}{K^2}. \quad (17)$$

Finding r_0' and φ_0 from Eq. (16) and substituting it in Eq. (3), we obtain

$$d^2j=\left(\frac{K}{\sin Kz}\right)^{-2}\times f\left\{r_0,\frac{K}{\sin Kz}\left[r_0\cos Kz-r\cos\left(\theta+Kz\right)\right],\ \frac{K}{\sin Kz}\left[r\sin\left(\theta+Kz\right)-r_0\sin Kz\right]\right\}. \quad (18)$$

As an example, we will consider the flow with the phase density at the starting section:

$$f(r_0,r_0',\varphi_0)=\begin{cases}\left(1+\frac{r_0'^2}{\sigma^2}+\frac{\varphi_0^2}{\sigma^2}\right)^{-2} & at\ r\le b,\\ 0 & at\ r>b,\end{cases} \quad (19)$$

to which Eq.(10) corresponds at $a\to\infty$. Then from Eq. (18) we have

$$d^2j=\frac{K^2}{\sin^2Kz}r_0dr_0d\theta\times\left[1+\frac{K^2}{\sigma^2\sin^2Kz}\left(r_0^2+r^2-2rr_0\cos\theta\right)\right]^{-2}, \quad (20)$$

and after integration with respect to θ and r_0 we will obtain:

$$j=\frac{\pi\sigma^2}{2}\left\{\frac{K^2\left(b^2-r^2\right)/\sigma^2\sin^2Kz-1}{C}+1\right\}, \quad (21)$$

where

$$C=\sqrt{\left(1+\frac{K^2b^2}{\sigma^2\sin^2Kz}\right)^2+\frac{2K^2r^2}{\sigma^2\sin^2Kz}\left(1-\frac{K^2b^2}{\sigma^2\sin^2Kz}\right)+\frac{K^4r^4}{\sigma^4\sin^4Kz}}.$$

The total current equals $I=\pi^2b^2\sigma^2$, and the envelope has the form:

$$R(z)=\sqrt{\kappa}\sqrt{b^2+\frac{\sigma^2\sin^2Kz}{(1-\kappa)K^2}}. \quad (22)$$

It is not difficult to see that at $K\to 0$ (no magnetic field) Eq. (21) and (22) turn into Eq. (14) and (15), respectively.

As one would expect, beam parameters are repeated with a periodicity $z=\pi/K$. The envelope is a sinusoid biased along the axis r with $R_{min}=\sqrt{\kappa}b$ and $R_{max}=\sqrt{\kappa}\sqrt{b^2+\frac{\sigma^2}{(1-\kappa)K^2}}$.

Thus, the method allowing it to calculate parameters of the axially symmetric beam (spatial distribution of the current density, beam envelope) from a known phase density in the orificing plane has been stated. The method's capabilities of calculation of beam transportation in free space and uniform magnetic field have been demonstrated. The examples given may be used for testing of numerical programs at the stage of adjustment. The constancy of a total current between membranes can serve as an extra criterion of the program's validity.

In closing a few remarks should be made. The method's capabilities are somewhat wider than those presented in the work. Actually, apertures in the orifice, as mentioned above, may have an arbitrary form. Thereat, the problem may prove to be not axially symmetric, which, however, has no effect on capabili-

ties of numerical methods of calculation.

Apart from the above statement of the problem, an inverse statement is also possible: to reconstruct the phase density of the beam in the starting plane from results of membranes burning-through. But it is another, more complicated problem not considered here.

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